

Lower energy consequences of an anomalous high-energy neutrino cross section

Haim Goldberg

Department of Physics, Northeastern University, Boston, Massachusetts 02115

T. J. Weiler

Department of Physics & Astronomy, Vanderbilt University, Nashville, Tennessee 37235

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A new strong interaction has been postulated for neutrinos above $\sim 10^{19}$ eV to explain the production of highest-energy cosmic-ray events. We derive a dispersion relation relating the hypothesized high-energy cross section to the lower-energy neutrino-nucleon elastic amplitude. Remarkably, we find that the real forward amplitude becomes anomalous seven orders of magnitude lower in energy than does the total cross section. We discuss possible measurable consequences of this early onset of new neutrino physics and conclude that a significantly enhanced elastic νN scattering rate may occur for the neutrino beams available at Fermilab and CERN. [S0556-2821(99)03709-1]

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I. INTRODUCTION

The discoveries by the Akemo Giant Air Shower Array (AGASA) [1], Fly's Eye [2], Haverah Park [3], and Yakutsk [4] Collaborations of air shower events with energies above the Greisen-Zatsepin-Kuzmin (GZK) cutoff of $\sim 5 \times 10^{19}$ eV challenge the standard model (SM) of particle physics and the hot big-bang model of cosmology. Not only is the mechanism for particle acceleration to such extremely high-energy cosmic rays (EHECRs) controversial, but also the propagation of EHECRs over cosmic distances is problematic. Above the resonant threshold for Δ^* production, $\sim 5 \times 10^{19}$ eV, protons lose energy by the scattering process $p + \gamma_{2.7\text{ K}} \rightarrow \Delta^* \rightarrow N + \pi$; $\gamma_{2.7\text{ K}}$ denotes a photon in the 2.7 K cosmic background radiation. This is the mechanism for the much-heralded GZK cutoff [5]. For every mean free path ~ 6 Mpc of travel, the proton loses 20% of its energy on average [6]. A proton produced at its cosmic source a distance D away will on average arrive at Earth with only a fraction $\sim (0.8)^{D/6\text{ Mpc}}$ of its original energy. Of course, proton energy is not lost significantly if the highest-energy protons come from a rather nearby source, $\lesssim 50\text{--}100$ Mpc.¹ However, acceleration of protons to $\sim 10^{20}$ eV, if possible at all, is generally believed to require the most extremely energetic compact sources, such as active galactic nuclei (AGNs) [8] or gamma-ray bursts (GRBs) [9]. Since AGNs and GRBs are hundreds of megaparsecs away, the energy requirement at an AGN or GRB for a proton which arrives at Earth with a super-GZK energy is unrealistically high [10]. A primary nucleus mitigates the cutoff problem (energy per nucleon is reduced by $1/A$), but above $\sim 10^{19}$ eV nuclei should be photo dissociated by the 2.7 K background [11] and possibly disintegrated by the particle density ambient at the astro-

physical source. Gamma rays and neutrinos are other possible primary candidates for the highest-energy events. The mean free path, however, for a $\sim 10^{20}$ eV photon to annihilate on the radio background to e^+e^- is believed to be only 10–40 Mpc [6].

Concerning the neutrino hypothesis, the Fly's Eye event occurred high in the atmosphere, whereas the event rate expected in the SM for early development of a neutrino-induced air shower is down from that of an electromagnetic or hadronic interaction by six orders of magnitude [12]. On the other hand, there is evidence that the arrival directions of some of the highest-energy primaries are paired with the directions of events lower in energy by an order of magnitude and displaced in time by about a year [13]. As noted in [13], such event pairing argues for stable neutral primaries coming from a source of considerable duration. Neutrino primaries do satisfy this criterion. Furthermore, a recent analysis of the arrival directions of the super-GZK events offers a tentative claim of a correlation with the directions of radio-loud quasars [14]. If this correlation is validated with future data, then the propagating cosmic particle must be charge neutral and have a negligible magnetic moment. The neutrino again emerges as the only candidate among the known particles. Thus it is of some interest to examine the possibility that the primary cosmic rays above the GZK cutoff energy are neutrinos, but with some large non-SM cross section that allows them to initiate air showers high in the atmosphere [15]. To mimic hadronically induced air showers, the new neutrino cross section must be of hadronic strength, ~ 100 mb, above $E_{\text{GZK}} \equiv 5 \times 10^{19}$ eV.

The use of an anomalously large high-energy neutrino interaction to model the observed super-GZK events has been criticized on the grounds that the onset of new neutrino physics cannot be so rapid as to hide the new interactions from experimental view at lower energies [16]. In turn, the criticism of the model has itself been criticized [17], on the grounds that a negative conclusion was drawn from simple perturbative calculations of single scalar or vector exchange models. In the present paper we analyze the hypothesized rapid rise in the neutrino cross section using an approach which is completely model independent. We assume that the

¹The suggestion has been made that hot spots of radio galaxies in the supergalactic plane at distances of tens of megaparsecs may be the sources of the super-GZK primaries [7]. Present statistics are too limited to validate or invalidate this proposal.

hypothesized new physics, whatever it may be, holds at and above an energy scale E^* , and use dispersion relations to provide a rigorous nonperturbative calculation of the growth of the *elastic* neutrino amplitude at much lower energies. We find that if new physics dominates the neutrino total cross section with a hadronic value σ^* above the laboratory energy E^* , then the real part of the new strong-interaction elastic amplitude at lower energy E is given by $(1/2\pi)(E/E^*)\sigma^*$. Thus, the new physics has the possibility of inducing a significant anomalous contribution to low-energy neutrino propagation in matter and to low-energy *elastic* neutrino-nucleon scattering.

The plan of the paper is as follows: in the next section we examine the implication for the low-energy elastic νN amplitude of an anomalous νN cross section at very high energies. In two subsections we discuss the experimental possibilities for testing this hypothesis. We calculate the anomalous index of refraction and effective potential induced for lower-energy neutrinos in matter and find that they are probably not measurable. We compute the enhanced elastic νN scattering cross section at lower energies and find that it may be measurable with neutrino beams existing at present accelerators. Section III contains some discussions and questions for further study, and our conclusions. The dispersion relations for the relevant νN amplitudes, central to this paper, are derived in an Appendix.

II. LOW-ENERGY ELASTIC AMPLITUDE FROM HIGH-ENERGY THRESHOLD

Suppose that there is a new neutrino-nucleon interaction of hadronic strength at neutrino laboratory energy $E' \geq E^*$ (i.e., $\sqrt{s'} \geq \sqrt{2ME^*}$), hypothesized to explain the air showers observed above the GZK cutoff. Then, for $E' \geq E^*$, we have

$$\sigma_{\text{tot}}^{\nu N}(E', \pm) = \sigma^*. \quad (1)$$

To address the question as to whether there are anomalous neutrino interactions below E^* , we invoke the dispersion relation for neutrino-nucleon scattering derived in the Appendix:

$$\begin{aligned} \text{Re } A_{\pm}(E) - \text{Re } A_{\pm}(0) \\ = \frac{E}{4\pi} \mathcal{P} \int_0^\infty dE' \left(\frac{\sigma_{\text{tot}}^{\nu N}(E', \pm)}{E'(E' - E)} + \frac{\sigma_{\text{tot}}^{\bar{\nu} N}(E', \pm)}{E'(E' + E)} \right), \end{aligned} \quad (2)$$

where $A_{\pm}(E)$ are invariant ν - N amplitudes, labeled by the nucleon helicity and defined in Eqs. (A1) and (A2) of the Appendix, and \mathcal{P} denotes the principle value of the integral. The new physics dominates the neutrino-nucleon dispersion integral (2) for $E' \geq E^*$. Motivated by simplicity and the behavior of the SM strong interaction, let us assume that σ^* is independent of helicity and energy, and that the new hadronic component of the neutrino cross section obeys the Pommeranchuk theorem

$$\sigma_{\text{tot}}^{\nu N}(E, \pm) - \sigma_{\text{tot}}^{\bar{\nu} N}(E, \pm) \xrightarrow{\nu \rightarrow \infty} 0. \quad (3)$$

These assumptions and the dispersion relation lead directly to an evaluation of the difference $\text{Re } A_{\pm}(E) - \text{Re } A_{\pm}(0)$, which is not calculable in perturbation theory. Ignoring ordinary weak-interaction contributions to the dispersion integral (which amounts to the omission of electroweak radiative corrections), we find, for the real part of the amplitude at energy E ,

$$\text{Re } A_{\pm}(E) \simeq \text{Re } A_{\pm}(0) + \Delta(E),$$

$$\Delta(E) \equiv \frac{1}{2\pi} \frac{E}{E^*} \sigma^*. \quad (4)$$

To arrive at simpler expressions below and to take advantage of the assumed helicity independence of the new interaction, it is convenient to work with the vector and axial vector amplitudes

$$C_V = \frac{1}{2}(A_- + A_+),$$

$$C_A = \frac{1}{2}(A_- - A_+). \quad (5)$$

From Eqs. (4), we have (again ignoring weak contributions to the dispersion integral)

$$\text{Re } C_V(E) = \text{Re } C_V(0) + \Delta(E),$$

$$\text{Re } C_A(E) = \text{Re } C_A(0). \quad (6)$$

The elastic amplitudes at $E=0$ are determined from Z exchange and are given by

$$C_V(0) = -\frac{G_F}{\sqrt{2}} [T_3(N) - 2 \sin^2 \theta_w Q(N)],$$

$$C_A(0) = -\frac{G_F}{\sqrt{2}} T_3(N), \quad (7)$$

with $N=p$ or n . (We have not explicitly shown the 20% renormalization of the axial vector amplitude due to QCD effects, since this percentage factor does not significantly affect what follows.) It will be convenient to use as a measure of the anomalous contribution the dimensionless ratio

$$\tilde{\Delta}(E) \equiv \frac{\Delta(E)}{G_F/2\sqrt{2}} \sim \mathcal{O}\left(\frac{\Delta(E)}{\text{Re } C_V(0)}\right). \quad (8)$$

Inputting Eqs. (4) and the numerical value of G_F into Eq. (8) gives the useful result

$$\tilde{\Delta}(E) \simeq \left(\frac{E/100 \text{ GeV}}{E^*/10^{18} \text{ eV}} \right) \left(\frac{\sigma^*}{100 \text{ mb}} \right). \quad (9)$$

It is clear from Eq. (9), and somewhat remarkable, that order 100% effects in the real elastic amplitudes begin to appear already at energies seven orders of magnitude below the full

realization of the strong cross section. This is our main physics result, from which we obtain the observable consequences discussed next.

A. Anomalous neutrino index of refraction and effective potential

The consequence of our neutrino dispersion relation and the sensible assumptions made for the new high-energy neutrino interaction is the anomalous real elastic amplitude at lower energies given in Eq. (4). The fractional increase in the real amplitude, compared to the SM value, is given in Eq. (9). Since the real part of the forward amplitude makes a direct contribution to the index of refraction n_{ref} , the most direct test of an anomalously large neutrino cross section would be a measurement of this refractive index. The real part of a forward amplitude is related to the refractive index n_{ref} by

$$n_{\text{ref}, \pm} - 1 = \frac{2\rho}{E} \text{Re } A_{\pm}(E), \quad (10)$$

where ρ is the nucleon number density of the (possibly polarized) medium. The anomalous contribution to the right-hand side of Eq. (10) exceeds the SM contribution at neutrino energies $E \gtrsim 100$ GeV. Perhaps fortuitously, 100 GeV is roughly the mean energy of atmospheric neutrinos producing thoroughgoing muons in underground detectors. According to SM physics, neutrinos at $E = 100$ GeV with mass-squared splittings $\lesssim 10^{-2}$ eV² receive significant phase contributions from matter effects and so would receive even larger effects from the new interaction. With $E \sim 100$ GeV and $\delta m^2 \sim 10^{-2}$ eV², there could be sizable new matter effects on oscillations over a distance of the order of the Earth diameter. However, if the anomalous reactions (if they exist) are flavor neutral, they produce a common phase and there will be no new matter effects associated with them.

B. Low-energy elastic scattering

There are more promising observable consequences available from the elastic cross section, obtained from the square of the elastic amplitude. The normalization of the amplitudes is such that the spin-averaged elastic scattering cross section in the forward direction is given by

$$\left. \frac{d\sigma}{dt} \right|_{t=0} = \frac{1}{\pi} [|C_V(E)|^2 + |C_A(E)|^2]. \quad (11)$$

In order to simplify the following discussion, let us assume that the elastic form factors of the nucleon effectively cut off at some common effective $|t| \sim m_{\text{eff}}^2$ and that the elastic amplitude is approximately real for $|t| \lesssim m_{\text{eff}}^2$. Then we may approximate the spin-averaged neutrino-nucleon elastic cross section as

$$\sigma_{\text{el}}^{\nu N}(E) \simeq \frac{m_{\text{eff}}^2}{\pi} \{[\text{Re } C_V(E)]^2 + [\text{Re } C_A(E)]^2\}. \quad (12)$$

It is useful now to frame the discussion in terms of the cross section normalized to the SM value:

$$\tilde{\sigma}_N \equiv \frac{\sigma_{\text{el}}^{\nu N}(E)}{\sigma_{\text{el}}^{\nu N}(E)|_{Z \text{ exchange}}} = \frac{[\tilde{\Delta} + a_N]^2 + 1}{a_N^2 + 1}, \quad (13)$$

with

$$a_N = 4 \sin^2 \theta_W Q(N) - \text{sgn}[T_3(N)] \\ = \begin{cases} 4 \sin^2 \theta_W - 1 & \text{for } N = p, \\ 1 & \text{for } N = n. \end{cases} \quad (14)$$

Use has been made of Eqs. (6), (7), (8), and (12) in arriving at Eqs. (13) and (14). With $\sin^2 \theta_W = 0.23$, one finds

$$\tilde{\sigma}_p \simeq \frac{(\tilde{\Delta} - 0.080)^2 + 1}{(0.080)^2 + 1}, \\ \tilde{\sigma}_n \simeq \frac{(\tilde{\Delta} + 1)^2 + 1}{2}. \quad (15)$$

From Eq. (15), it is apparent that a significant enhancement in the elastic cross sections (say, a factor of 10 or more) is obtained for $\tilde{\Delta} > 3$. It is encouraging that such a value of $\tilde{\Delta}$ may be within reach of current experimental setups, rather than orders of magnitude beyond. We can see from Eq. (9) that the anomalous elastic scattering cross section is already significant at energy E related to the energy E^* characterizing the anomalous inelastic scattering by

$$E \simeq \left(\frac{\tilde{\Delta}(E)}{3} \right) \left(\frac{100 \text{ mb}}{\sigma^*} \right) \left(\frac{E^*}{10^{17.5} \text{ eV}} \right) \times 100 \text{ GeV}. \quad (16)$$

This result says that if the neutrino is strongly interacting at $E^* \sim 10^{17.5}$ eV then an anomalous rise in the elastic cross section is occurring at neutrino energies already available at existing accelerators. The Fermilab neutrino beam used by the NuTeV experiment has a mean energy of about 100 GeV. The CERN neutrino beam used in oscillation experiments contains neutrinos at 100 GeV, with an intensity down by a factor of ~ 20 from the intensity at the mean energy of 30 GeV.

We propose that a comparison of the elastic event rate below and above 100 GeV be done, to look for the onset of anomalous neutrino elastic scattering. The proposed experiment is difficult, for the identification of elastic neutrino scattering is challenging. A low-energy recoil proton must be detected, with a veto on events with pions produced. Because the momentum transfer in elastic scattering is limited to $\lesssim 1$ GeV², the recoil nucleon has a kinetic energy of at most 0.5 GeV.

We end this section by noting that since the ratios $\tilde{\sigma}_{p,n}$ grow quadratically with E , anomalies in the elastic cross section develop rapidly for $E > 100$ GeV. Thus the event sample of a future underground/water/ice neutrino telescope optimized for TeV neutrinos could conceivably contain 1000

times more elastic neutrino events than predicted by the SM, and a telescope optimized for PeV neutrinos may contain 10^9 more elastic events.

III. DISCUSSION AND CONCLUSIONS

The hypothesized new strong interaction for neutrinos with energy above E_{GZK} is extraordinarily speculative. The only phenomenological argument for the hypothesis is that it provides a possible explanation of the super-GZK events. Yet it may be testable at lower energies because it implies, as we have shown, an anomalous contribution to the real amplitude at lower energies. The dispersion-relation formalism presented here provides a model-independent theoretical framework for examining the lower-energy implications of the high-energy neutrino strong-interaction hypothesis.

We have discussed two classes of low-energy tests of the neutrino strong-interaction hypothesis. The first uses the real amplitude directly to calculate an anomalous neutrino index of refraction, resulting in possible anomalous matter effects for flavor oscillations. This approach is viable only if there is flavor dependence in the anomalous interaction. The other class uses the squared amplitude to calculate anomalous neutrino-nucleon elastic scattering. With elastic scattering, the hypothesis may be testable already using 100 GeV neutrinos, even though the strong-interaction inelastic cross section does not develop until near E_{GZK} .

We have seen that a measurable anomalous elastic signal at 100 GeV requires a value of $10^{17.5}$ eV for E^* , characterizing the high-energy anomaly. Is a value of E^* as low as this possible? For now, the data do not rule out such a possibility. The break in the very-high-energy cosmic ray data (the ‘‘ankle’’) seems to occur at an energy of $10^{18.5}$ eV, where it is thought that the transition between galactic and extragalactic sources is evolving. This is not inconsistent with neutrinos attaining their hadronic cross sections an order of magnitude lower in energy, but not yet dominating the cosmic-ray spectrum. This picture gains some support from a two-component fit to the Fly’s Eye data [2], which suggests that the extragalactic high-energy spectrum begins to appear already at about $10^{17.5}$ eV.

There may be other tests of the strong-interaction hypothesis beyond those formulated here. For example, if the neutrino develops a strong interaction at high energy, do not the electron and the other charged-lepton SU (2)-doublet partners of the neutrinos also develop a similar strong interaction? If so, is there new physics to be sought in charged-lepton–nucleon scattering in the highest-energy cosmic-ray air showers? Is there new physics in the elastic $e^\pm p$ scattering channel at energies reached at the DESY ep collider HERA?² If elastic scattering is enhanced at lower energies, is it not likely that quasielastic scattering is also enhanced? If

so, the anomalous event rate could differ significantly from what we have calculated from just the elastic channel.³

Our proposal to look for the onset of an anomalous enhancement in the elastic scattering rate around 100 GeV, using presently available neutrino beams, is a conservative first step toward empirically answering these questions.

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APPENDIX: DERIVATION OF NEUTRINO DISPERSION RELATIONS

Consider the elastic scattering of a (left-handed) neutrino, incident along the $+z$ axis, from a nucleon N whose mass is M . The S matrix can be written as

$$S_{fi} = 1 + (2\pi)^4 i \delta^4(P_f - P_i) T(k', p', \lambda'; k, p, \lambda), \quad (\text{A1})$$

where

$$\begin{aligned} T(k', p', \lambda'; k, p, \lambda) &= \frac{1}{(2E_2 E' 2\omega 2\omega')^{1/2}} \mathcal{M}(k', p', \lambda'; k, p, \lambda), \\ \mathcal{M}(k', p', \lambda'; k, p, \lambda) &= \bar{u}_\nu(k') \gamma^\mu (1 - \gamma_5) u_\nu(k) \\ &\quad \times \bar{u}_N(p', \lambda') \gamma_\mu [A_-(s, t) L + A_+(s, t) R] u_N(p, \lambda) \end{aligned} \quad (\text{A2})$$

with λ, λ' labeling the initial and final nucleon helicities, and the projection operators $L, R = (1 \mp \gamma_5)/2$; volume factors in the normalization of T are omitted. The Mandelstam variables are defined as usual:

$$\begin{aligned} s &= (k + p)^2 = (k' + p')^2, \\ t &= (k - k')^2 = (p - p')^2, \\ u &= (p - k')^2 = (p' - k)^2, \end{aligned} \quad (\text{A3})$$

with $s + t + u = 2M^2 + 2m_\nu^2$. (We retain a small neutrino mass m_ν for the moment.)

The optical theorem relating the forward amplitude to the total cross section (for a fixed initial nucleon spin) reads

$$\text{Im } \mathcal{M}(k, p, \lambda; k, p, \lambda) = 2M \sqrt{E^2 - m_\nu^2} \sigma_{\text{tot}}^{\nu N}(E, \lambda), \quad (\text{A4})$$

²However, in the elastic $e^\pm p$ channel at HERA, a simple calculation shows that one-photon exchange dominates the usual neutral current interactions at small momentum transfer to such an extent that even at the effective laboratory energy of $\sim 10^5$ GeV applicable to HERA the anomalous scattering contribution is at the 1% level.

³A possible enhancement in the quasielastic channel cannot be deduced from dispersion relations. A separate calculation could be made, in principle, if the details of the new high-energy strong interaction are specified.

where E is the neutrino laboratory energy and λ is now the nucleon spin along the z axis. A little bit of algebra shows that, for forward scattering,

$$\mathcal{M}(k, p, \pm; k, p, \pm) = 8ME A_{\pm}(s, 0). \quad (\text{A5})$$

It will prove convenient to work in terms of the invariant quantity ν , defined by

$$\nu \equiv (p + p') \cdot (k + k') / (4M) = (s - u) / (4M). \quad (\text{A6})$$

For forward elastic scattering of a neutrino of laboratory energy E on a stationary target of mass M , $\nu = E$.

Ignoring subtractions for the moment, the analytic property of A_{\pm} is expressed through the Hilbert transform in the ν plane (for fixed t)

$$A_{\pm}(\nu, t) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{d\nu' \text{Im} A_{\pm}(\nu', t)}{\nu' - \nu - i\epsilon}. \quad (\text{A7})$$

There are two cuts.

(1) s -channel cut: from $s = (M + m_{\nu})^2$ to $s = \infty$ from physical νN scattering. In the ν plane this gives a cut from $\nu_{\text{th}} = m_{\nu} + t/(4M)$ to $\nu = \infty$.

(2) u -channel cut: from $u = (M + m_{\nu})^2$ to $u = \infty$ from physical $\bar{\nu}N$ scattering. Substituting $s = 2M^2 + 2m_{\nu}^2 - u - t$ into Eq. (A6), we find that in the ν plane the u -channel cut extends from $\nu = -\infty$ to $\nu = -\nu_{\text{th}}$.

Thus Eq. (A7) becomes

$$A_{\pm}(\nu, t) = \frac{1}{\pi} \int_{-\infty}^{-\nu_{\text{th}}} \frac{d\nu' \text{Im} A_{\pm}(\nu', t)}{\nu' - \nu - i\epsilon} + \frac{1}{\pi} \int_{\nu_{\text{th}}}^{\infty} \frac{d\nu' \text{Im} A_{\pm}(\nu', t)}{\nu' - \nu - i\epsilon}. \quad (\text{A8})$$

With a change in variables $\nu' \rightarrow -\nu'$ in the second integral, Eq. (A8) becomes

$$A_{\pm}(\nu, t) = \frac{1}{\pi} \int_{\nu_{\text{th}}}^{\infty} \frac{d\nu' \text{Im} A_{\pm}(\nu', t)}{\nu' - \nu - i\epsilon} - \frac{1}{\pi} \int_{\nu_{\text{th}}}^{\infty} \frac{d\nu' \text{Im} A_{\pm}(-\nu', t)}{\nu' + \nu + i\epsilon}. \quad (\text{A9})$$

We now use crossing. First, define an amplitude for $\bar{\nu}N$ scattering in a manner analogous to Eq. (A2):

$$\begin{aligned} \bar{\mathcal{M}}(k', p', \lambda'; k, p, \lambda) \\ = \bar{v}_{\nu}(k) \gamma^{\mu} (1 - \gamma_5) v_{\nu}(k') \\ \times \bar{u}_N(p', \lambda') \gamma_{\mu} [\bar{A}_{-}(\nu, t) L + \bar{A}_{+}(\nu, t) R] u_N(p, \lambda). \end{aligned} \quad (\text{A10})$$

It is a lengthy but straightforward exercise to derive the crossing relation between A_{\pm} and the analogous amplitude for the right-handed antineutrino, \bar{A}_{\pm} . The method used is essentially that which can be found in [18], and can also be

checked by constructing an effective Lagrangian which will yield the amplitude (A2) at the tree level. The result is

$$\text{Re} \bar{A}_{\pm}(\nu, t) = -\text{Re} A_{\pm}(-\nu, t). \quad (\text{A11})$$

The minus sign comes from the anticommuting properties of fermion operators, which come into play when the antineutrino in- and out-field operators are brought into normal order. By writing a dispersion relation for \bar{A}_{\pm} similar to Eq. (A9) and comparing to the latter using Eq. (A11), one discovers that consistency requires

$$\text{Im} \bar{A}_{\pm}(\nu, t) = +\text{Im} A_{\pm}(-\nu, t). \quad (\text{A12})$$

Substituting Eq. (A12) into (A9), one obtains

$$\text{Re} A_{\pm}(\nu, t) = \frac{1}{\pi} \mathcal{P} \int_{\nu_{\text{th}}}^{\infty} d\nu' \left(\frac{\text{Im} A_{\pm}(\nu', t)}{\nu' - \nu} - \frac{\text{Im} \bar{A}_{\pm}(\nu', t)}{\nu' + \nu} \right), \quad (\text{A13})$$

with \mathcal{P} signifying the principal value of the integral.

Now specialize to the forward direction ($t=0$) and to massless neutrinos (in which case, $\nu_{\text{th}}=0$). We will call the forward (invariant) scattering amplitude $A_{\pm}(E)$, reverting to the neutrino laboratory energy as the kinematic variable. With the use of Eqs. (A4) and (A5), the optical theorem in terms of the amplitude A_{\pm} reads

$$\text{Im} A_{\pm}(E) = \frac{1}{4} \sigma_{\text{tot}}^{\nu N}(E, \pm), \quad (\text{A14})$$

with a similar equation for \bar{A}_{\pm} :

$$\text{Im} \bar{A}_{\pm}(E) = \frac{1}{4} \sigma_{\text{tot}}^{\bar{\nu} N}(E, \pm). \quad (\text{A15})$$

After inserting Eqs. (A14) and (A15), the dispersion relation (A13) reads

$$\text{Re} A_{\pm}(E) = \frac{1}{4\pi} \mathcal{P} \int_0^{\infty} dE' \left(\frac{\sigma_{\text{tot}}^{\nu N}(E', \pm)}{E' - E} - \frac{\sigma_{\text{tot}}^{\bar{\nu} N}(E', \pm)}{E' + E} \right). \quad (\text{A16})$$

To improve the convergence of the integral,⁴ we rewrite Eq. (A16) in the once-subtracted form

$$\begin{aligned} \text{Re} A_{\pm}(E) - \text{Re} A_{\pm}(0) \\ = \frac{E}{4\pi} \mathcal{P} \int_0^{\infty} dE' \left(\frac{\sigma_{\text{tot}}^{\nu N}(E', \pm)}{E'(E' - E)} + \frac{\sigma_{\text{tot}}^{\bar{\nu} N}(E', \pm)}{E'(E' + E)} \right). \end{aligned} \quad (\text{A17})$$

No information is lost in the subtraction, since the subtraction constant $\text{Re} A_{\pm}(0)$ is known from the standard model. Equation (A17) provides the theoretical basis for the phenomenological considerations in the main text.

⁴Note that the Pomeranchuk theorem does *not* hold in the electroweak theory—if it did, the threshold amplitude $\text{Re} A_{\pm}(0)$ could be calculated from Eq. (A16).

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